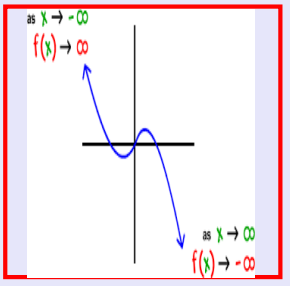
Math 245

Spring 2022

Lecture 14



Class QZ 5

1,-30
2,15
3,-10
5,-6

Factor Completely:

1) $x^2 - x - 30 = (x-6)(x+5)$ ✓

Trinomial

2) $x^2 - 36 = x^2 - 6^2 = (x-6)(x+6)$ ✓

$A^2 - B^2 = (A-B)(A+B)$

3) $x^3 + 8 = x^3 + 2^3 = (x+2)(x^2 - 2x + 4)$ ✓

$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$

Solve $2x^2 - 5 \geq 3x$

1) Make RHS=0 $\quad 2x^2 - 5 - 3x \geq 0$

2) LHS must be Comp. Factored $\quad 2x^2 - 3x - 5 \geq 0$

$(2x-5)(x+1) \geq 0$

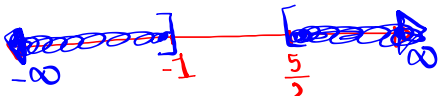
3) Set each factor =0, Solve $\quad 2x-5=0 \quad x+1=0$

$x = \frac{5}{2} \quad x = -1$

4) Sign chart

x	$-\infty$	-1	$\frac{5}{2}$	∞
$x+1$	$-$	0 $+$	$+$	$+$
$2x-5$	$-$	$-$	0 $+$	$+$
Product	$+$	0 $-$	0 $+$	

$x \leq -1$ $\qquad x \geq \frac{5}{2}$



I.N. $(-\infty, -1] \cup [\frac{5}{2}, \infty)$, S.B.N. $\{x \mid x \leq -1 \text{ or } x \geq \frac{5}{2}\}$

Solve $x^3 - 25x < 8x^2 - 200$

$x^3 - 25x - 8x^2 + 200 < 0$

$x^3 - 8x^2 - 25x + 200 < 0$ $\qquad$ RHS

$x^2(x-8) - 25(x-8) < 0$

$(x-8)(x^2-25) < 0 \Rightarrow (x-8)(x+5)(x-5) < 0$

LHS Factored

$x-8=0 \to x=8$

$x+5=0 \to x=-5$

$x-5=0 \to x=5$

x	$-\infty$	-5	5	8	∞
$x+5$	$-$	0 $+$	$+$	$+$	$+$
$x-5$	$-$	$-$	0 $+$	$+$	$+$
$x-8$	$-$	$-$	$-$	0 $+$	$+$
Problem	$-$	0 $+$	0 $-$	0 $+$	

$x < -5$ $\qquad 5 < x < 8$

Graphing

I.N. $(-\infty, -5) \cup (5, 8)$ $\qquad$ S.B.N. $\{x \mid x < -5 \text{ or } 5 < x < 8\}$

Solve $\frac{3}{x+8} \leq \frac{1}{x-4}$

RHS = 0 $\quad \frac{3}{x+8} - \frac{1}{x-4} \leq 0$

LHS must be a single fraction in factored form

$\frac{3(x-4)}{(x+8)(x-4)} - \frac{1(x+8)}{(x-4)(x+8)} \leq 0$

$\frac{3(x-4) - 1(x+8)}{(x+8)(x-4)} \leq 0$

$\frac{2x-20}{(x+8)(x-4)} \leq 0$

$\frac{2(x-10)}{(x+8)(x-4)} \leq 0$

$2 > 0 \rightarrow +$

$x-10=0 \rightarrow x=10$

$x+8=0 \rightarrow x=-8$

$x-4=0 \rightarrow x=4$

x	$-\infty$	-8	4	10	∞
2	+	+	+	+	
$x+8$	−	0	+	+	
$x-4$	−	−	0	+	
$x-10$	−	−	−	0	+
Problem	−	+	−	0	+

$x \leq -8$ $\quad$ $4 < x \leq 10$

I.N. $(-\infty, -8) \cup (4, 10]$

S.B.N. $\{x \mid x \leq -8 \text{ OR } 4 < x \leq 10\}$

Z varies directly as x^2 and inversely as $\sqrt{y}$.

Z is 30 when $x = 5$ and $y = 25$

$Z = \frac{Kx^2}{\sqrt{y}}$

$30 = \frac{K \cdot 5^2}{\sqrt{25}}$

$30 = \frac{K \cdot 25}{5}$

$30 = K \cdot 5$

$K = 6$

Find Z when x is 10 and y is 100.

$Z = \frac{6x^2}{\sqrt{y}}$

$Z = \frac{6 \cdot 10^2}{\sqrt{100}} = \frac{6 \cdot \cancel{100}^{10}}{\cancel{10}}$

$Z = 60$

Quadratic Equation $\Rightarrow$ $ax^2+bx+c=0$, $a\neq 0$

Quadratic Formula $\Rightarrow$ $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$b^2-4ac \Rightarrow$ Discriminant

Solve $2x^2-3x-5=0$ by Q-Formula.

$ax^2+bx+c=0$

$a=2$, $b=-3$, $c=-5$ $\Rightarrow$

$b^2-4ac=(-3)^2-4(2)(-5)=\boxed{49}$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-(-3)\pm\sqrt{49}}{2(2)}$

$=\dfrac{3\pm 7}{4}$

$x=\dfrac{3+7}{4}=\dfrac{10}{4}$, $\boxed{x=\dfrac{5}{2}}$

$x=\dfrac{3-7}{4}=\dfrac{-4}{4}$, $\boxed{x=-1}$

Solution Set $\left\{-1, \dfrac{5}{2}\right\}$

Solve $3x^2=8x-5$ by Quadratic Formula.

$3x^2-8x+5=0$

$ax^2+bx+c=0$

$a=3$, $b=-8$, $c=5$

$b^2-4ac=(-8)^2-4(3)(5)=4$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-(-8)\pm\sqrt{4}}{2(3)}=\dfrac{8\pm 2}{6}$

$x=\dfrac{8+2}{6}=\dfrac{10}{6}=\boxed{\dfrac{5}{3}}$

$x=\dfrac{8-2}{6}=\dfrac{6}{6}=\boxed{1}$

Solution Set $\left\{1, \dfrac{5}{3}\right\}$